P P SAVANI UNIVERSITY

First Semester of M.Sc. DS Examination

February 2023

SESH7020 Mathematical Methods for Data Science

13.02.2023, Monday **Time: 10:00 a.m. To 12:30 p.m.** **Maximum Marks: 60**

Instructions:

1. The question paper comprises of two sections.
2. Section I and II must be attempted in separate answer sheets.
3. Make suitable assumptions and draw neat figures wherever required.
4. Use of scientific calculator is allowed.

SECTION – I

Q - 1	Choose the correct answer.	**[05]**	CO	BTL
(i)	The n^{th} derivative of e^{ax} is ____. (a) e^{ax} (b) $a^x e^{ax}$ (c) e^{anx} (d) $a^n e^{ax}$		1	1
(ii)	The value of $\lim\limits_{x\to 1}\frac{x^2+x-2}{x^2-x}$ is ____. (a) 3 (b) 2 (c) 1 (d) 0		1	2,3
(iii)	The value of $\lim\limits_{n\to\infty}\frac{\log n}{n}=$ ____. (a) ∞ (b) 0 (c) 1 (d) e		1	2,3
(iv)	A function $f(x,y)$ is said to be homogeneous function in which the power of each term is the ____. (a) same (b) different (c) both (a) and (b) (d) None		2	2
(v)	The second derivative test if $f_{xx}f_{yy}-f_{xy}{}^2=0$ at (a,b) then ____. (a) maximum (b) minimum (c) inconclusive (d) saddle point		4	2
Q - 2 (a)	Check whether the function is continuous or not: $f(x)=\begin{cases}\frac{\sin x}{x} & ;x\neq 0\\ 0 & ;x=0\end{cases}$. Identify types of discontinuity.	**[05]**	1	4,5
Q - 2 (b)	Verify Rolle's theorem in $[-1,1]$ for function $f(x)=x^2$.	**[05]**	1	5
	OR			
Q - 2 (a)	Evaluate following limits: I. $\lim\limits_{x\to -1}\frac{x^3-x^2-5x-3}{(x+1)^2}$ II. $\lim\limits_{x\to 1}\frac{x^{-1}-1}{x-1}$	**[05]**	1	5
Q - 2 (b)	Test the convergence of the series $\frac{1}{1\cdot 3}+\frac{2}{3\cdot 5}+\frac{3}{5\cdot 7}+\frac{4}{7\cdot 9}+\cdots$.	**[05]**	2	4,5
Q - 3 (a)	Test the convergence of series $\sum\limits_{n=1}^{\infty}\frac{2^n}{n^3+1}$.	**[05]**	2	4,5
Q - 3 (b)	Find the values of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at the point $(4,-5)$ if $f(x,y)=x^2+3xy+y-1$.	**[05]**	4	5
	OR			
Q - 3 (a)	Find the Maclaurin's series expansion of $\cosh x$ and $\sinh x$.	**[05]**	2	3,5

Q - 3 (b)	If x, y and z are independent variables and $f(x,y,z)=x\sin(y+3z)$ then find $\frac{\partial f}{\partial z}$.	[05]	4	5
Q - 4	If $z=xf\left(\frac{y}{x}\right)+g\left(\frac{y}{x}\right)$, show that $x^2\frac{\partial^2 z}{\partial x^2}+2xy\frac{\partial^2 z}{\partial x\partial y}+y^2\frac{\partial^2 z}{\partial y^2}=0$.	[05]	4	3,5

SECTION – II

Q - 1	Choose the correct answer.	[05]		
(i)	$\dim(P_n)=$____. (a) n (b) $n+1$ (c) $n-1$ (d) 0		3	2
(ii)	The vector space $V=0$ is called ____ vector space. (a) trivial (b) non-trivial (c) closed (d) None		3	1
(iii)	$r(T)+n(T)=$____. (a) $range(T)$ (b) $\ker(T)$ (c) $\dim(V)$ (d) None		3	1
(iv)	A vector of norm 1 is called a ____ vector. (a) single (b) zero (c) one (d) unit		3	1
(v)	If u and v are orthogonal vectors in a real inner product space, then $\lVert u+v\rVert^2=$____. (a) $\lVert u^2+v^2\rVert$ (b) $\lVert u\rVert\cdot\lVert v\rVert$ (c) $\lVert u\rVert^2+\lVert v\rVert^2$ (d) $\lVert u^2v^2\rVert$		3	2
Q - 2	Let V be the set of all 2×2 matrices of the form $\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix}$ with addition defined by $\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix}+\begin{bmatrix} c & 1 \\ 1 & d \end{bmatrix}=\begin{bmatrix} a+c & 1 \\ 1 & b+d \end{bmatrix}$ and scalar multiplication $k\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix}=\begin{bmatrix} ka & 1 \\ 1 & kb \end{bmatrix}$. Prove that V is a vector space.	[10]	3	4,5
	OR			
Q - 2 (a)	Find the dimension of the solution space of the LS $x_1+x_2-x_3+x_4=0$; $-x_2-2x_3-x_4=0$.	[05]	3	4
Q - 2 (b)	Show that the vectors $v_1=(1,2,1)$, $v_2=(2,9,0)$ and $v_3=(3,3,4)$ form a basis for R^3.	[05]	3	4
Q - 3 (a)	Consider the transformation $T:R^3\to R^2$, defined by $T(x)=(x_1-2x_2+x_3, x_2+5x_3)$. Is T a linear transformation?	[05]	3	4,6
Q - 3 (b)	Let $T:M_{22}\to R^2$ defined by $T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right)=\begin{bmatrix} a+b \\ c+d \end{bmatrix}$ is linear. Describe its kernel and range.	[05]	3	3,5
	OR			
Q - 3 (a)	Let $u=(u_1,u_2)$ and $v=(v_1,v_2)$ be vectors in R^2. Verify that the weighted Euclidean inner product $\langle u,v\rangle=3u_1v_1+2u_2v_2$ satisfies the four inner product axioms.	[05]	3	3,4
Q - 3 (b)	Let $u=(1,3,-4,2)$ and $v=(4,-2,2,1)$ in R^4. Find the normalize of u and v.	[05]	3	5

Q - 4 Find a QR-decomposition of $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$. [05] 3 5

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CO : Course Outcome Number BTL : Blooms Taxonomy Level

Level of Bloom's Revised Taxonomy in Assessment

1: Remember	2: Understand	3: Apply
4: Analyze	5: Evaluate	6: Create